CHAPTER 2.3

Practising the correspondence principle in the old quantum theory: Franck, Hund and the Ramsauer effect

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Abstract

The paper discusses the work of James Franck and Friedrich Hund on the Ramsauer effect as an example for the reception and adaptation of the correspondence principle in the old quantum theory. In their attempt to account for the Ramsauer effect, Franck and Hund incorporated the correspondence principle into their description of electron scattering and encountered several problems in the correspondence approach. In reaction to these challenges, Franck and Hund adapted the principle and developed both a new interpretation of the correspondence principle and a new understanding of scattering processes.

Key words: Correspondence principle; reception; application; Ramsauer effect; James Franck; Friedrich Hund.

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1. Introduction: The dispersion and adaptation of the correspondence principle

In 1922 Niels Bohr was frustrated with the reception of his work on the quantum theory of the atom. Complaining to Arnold Sommerfeld, he wrote: "[i]n the last years ... my attempts to develop the principles of quantum theory ... were met with very little understanding."[1] Thereby he diagnosed that his foundational ideas – most importantly the correspondence principle[2] – had not played an important role in the rapid developments of the quantum theory of the atom outside of Copenhagen.

Physicists outside of Copenhagen had understood the core idea of the correspondence principle in a remarkably homogenous way: the principle, they knew, connected the Fourier-representation of the radiating system with its radiation spectrum. By the time, however, they took little more from this core idea than the justification of selection rules, which Bohr had presented in his article "On the Quantum Theory of Line Spectra."[3] The situation only changed from 1922 onwards. More and more physicists working in Munich, Göttingen, Breslau, Harvard, Minnesota etc. started to explore the significance of the principle for their work and to extend it to research fields ranging from atomic and molecular spectra to dispersion or collision processes.

The dissemination of the correspondence principle within the networks of quantum theory and its impact on the development of the correspondence principle have not received much attention in

1. Bohr to Sommerfeld 30 April 1922. Sommerfeld (2004), p. 117.

2. For reconstructions of Bohr's correspondence argument and its formulation, see Rud Nielsen (1976), Darrigol (1992), Tanona (2002), Bokulich (2013) and Kragh (2012), as well as the contribution to this volume by Robert Rynasiewicz. The most pronounced formulation was given by Bohr in his last grand survey article on the quantum theory of multiply periodic systems, Bohr (1923).

3. Bohr (1918a), Bohr (1918b). Compare the most widely received secondary accounts of the period: Reiche (1921), Landé (1922), Buchwald (1923), Sommerfeld (1924), Born (1925) and Pauli (1925). For an extended bibliography and discussion, see my forthcoming dissertation, Jähnert (forthcoming).

the historiography of quantum physics.[4] Studies of the principle have focussed mainly on Bohr's formulation or on the role of the principle in the development of quantum mechanics.[5] As I intend to show in my dissertation and hint at in this paper, these undoubtedly important topics are only the tip of the iceberg in a history of the correspondence principle. The part below the surface consists of many attempts to use and adapt the principle in the old quantum theory. Regardless of whether – in hindsight – these attempts have to be considered as important steps in the conceptual development of quantum physics or as short-lived, unsuccessful attempts at a singular problem, the study of the principle's applications sheds light on how a major part of theoretical physics was done in the period between 1922 and 1926.

Of course, this paper cannot present a fullblown discussion of the adaptation of the principle in the widely different theoretical, institutional and social contexts. Instead, I will focus on one particular case study and discuss the work of James Franck and Friedrich Hund on the Ramsauer effect as an example for the transfer and adaptation of the correspondence principle. This case study, which is based on previously unused archival material,[6] is particularly well suited to characterize some of the main features of the principle's *adaptive reformulation* and allows an illustration of the mechanisms of the spreading and adaptation of the correspondence principle at large. The analysis of this episode will allow three more general questions to be addressed: What led physicists to work with the correspondence principle in the first place? What did they do with it? And last but not least, what did the application of the prin-

4. The work of Duncan and Janssen (2007a,b) and Jordi Taltavull (2013) are among the few studies that analyze the role of the correspondence principle in one particular research field.

5. See footnote 2.

6. Especially Hund's unpublished dissertation, Hund (1922), and, most importantly, his scientific diary, Hund Papers, did not play a role in the secondary literature. A short discussion of Franck's and Hund's approach is given by Darrigol (1992), pp. 250-252. The only major historical study of the Ramsauer effect, Gyeong (1995), is dedicated to its experimental origins and leaves Franck's and Hund's theoretical discussions aside.

ciple mean for the understanding of physical problems and the formulation of the correspondence principle itself? As will be shown, the answers to these questions involve a transfer of ideas from Copenhagen to Göttingen, a conceptual reorganization of ideas exchanged between different research fields, several adaptations of the correspondence principle itself and the formulation of a new understanding of scattering processes.

2. From the *2π*-deflection hypothesis to the correspondence principle

When Franck's and Hund's work on collisions between electrons and gas atoms began in 1921, the two physicists approached the newly found Ramsauer effect within the framework of classical physics. Neither quantum theory in general nor the correspondence principle in particular seemed important for their work.[7] As I will discuss in this section, the principle only took center stage after discussions between Franck, Bohr and Kramers in Copenhagen. Following them, Franck and Hund integrated the correspondence approach into their work and developed a first quantum-theoretic interpretation of the Ramsauer effect.

The Ramsauer effect became a research topic for physicists in Göttingen following the "Deutsche Physikertage" held in Jena in September 1921, where Franck had witnessed a talk on the experiments on the passage of very slow electrons through noble gases by Carl Ramsauer. Ramsauer's results showed an unexpected relation between the velocity of slow electrons and the atomic cross section measuring the strength of the interaction between atoms and elec-

7. A general conviction that collisions between electrons and atoms had to be described quantum-theoretically did not exist at the time. The only prominent example concerning the relation between collision experiments and quantum theory was the Franck-Hertz experiment. It had to be described quantum-theoretically, as the kinetic energy of the electron is transferred to the atom that makes transitions between different stationary states. Elastic collisions – as in the Ramsauer effect – however, did not involve such an energy transfer, so that quantum assumptions did not play a role. For an overview on scattering theory, see Franck (1923) and Landé (1926), pp. 21-22.

trons. While the atomic cross section was constant for electrons of different velocities for neon and helium, it decreased considerably for very slow electrons passing through argon. Very slow electrons, it seemed, ceased interacting with the argon atoms and passed through the gas without any disturbance.

Franck was not willing to accept this implication and was therefore quite irritated by Ramsauer's results. If anything, very slow electrons should be strongly affected by atomic force fields and the atomic cross section had to increase rather than to decrease. The most likely explanation, he thought, was an experimental error on Ramsauer's part. Bringing the problem to the classroom of his *Proseminar* in Göttingen, he and Max Born set students to work on the experimental and theoretical refutation of "Ramsauer's crazy assertion."[8]

The Göttingen community passed the stage of denial in March 1922, after Franck's long-time friend and colleague Gustav Hertz had confirmed the strange behavior of electrons in argon.[9] Nonetheless, the idea that electrons ceased to interact with atoms remained unacceptable to Franck. As argon was the only gas known to show the effect, he considered the possibility of an anomalous interaction between the electrons and the argon atoms. Argon atoms, Franck speculated, might have a special field of force, which deflected slow electrons into an angle of 2π.[10] The electrons would go through one loop around the atom and then leave it as if no interaction occurred. The task of constructing such a force field was taken over by Friedrich Hund in his doctoral dissertation. Well prepared by his *Examensarbeit* on potential theory written with Richard Courant, he produced a force field for the desired 2π-deflection. Finishing his work on the "argon effect" in September 1922, Hund was

8. Ramsauer's talk was published as Ramsauer (1921a) and was followed by a series of papers, Ramsauer (1921b, 1922, 1923). For Franck's reception, see Franck to Bohr 25 September 1921, Bohr (1987), p. 689, and Born to Einstein 29 November 1921, Born (1969), pp. 91-93.

9. For the first indications of this confirmation, see Hertz to Franck 15 December 1921, Franck Papers, Box 3, Folder 13. See also Franck to Bohr 21 February 1922, Bohr (1987), p. 693, and Hertz (1922a,b).

10. Franck to Bohr 21 February 1922, Bohr (1987), p. 693.

ready to hand in his dissertation at the Georg-August-Universität in Göttingen. He would then work on Born's lecture course "Atommechanik" and the related research program on the quantum theory of atomic spectra as Born's new assistant.

As mentioned earlier, quantum hypotheses did not play a substantial role in Franck's and Hund's early considerations. During the *Bohr Festspiele* in June 1922, Hund briefly considered a combination of the 2π-deflection hypothesis with the idea of distinguishing quantum orbits from classically allowed ones. However, he quickly returned to the purely classical approach after his idea was criticized as "too formal" in discussions with "Bohr's assistants and the *Bonzen.*"[11] In October 1922, however, quantum theory and with it the correspondence principle took center stage in Franck's and Hund's work.

Franck developed a new interpretation of Ramsauer's experiments after discussions with Bohr and Kramers in Copenhagen, where he and his wife stayed with Niels and Margarethe Bohr at Bohr's new institute. Franck learned that Kramers was currently working on the continuous X-ray spectrum and its quantum-theoretical interpretation. The main idea of a forthcomming paper, Kramers informed him, was to extend the correspondence principle to aperiodic motions and to construct the continuous X-ray spectrum from a Fourier-analysis of the trajectory of a scattered electron.[12]

This argument, Franck recognized immediately, used the description of scattering that was also underlying his and Hund's work on the Ramsauer effect. Notwithstanding the fact that Kramers's work was concerned with an entirely different physical phenomenon – X-rays produced by electrons with an energy a thousand times higher than Ramsauer's slow electrons – Franck started to think about the implications of the correspondence approach for his own work. Following them, he formulated a new interpretation of the Ramsauer effect, which integrated the challenging facts regarding the continuous X-ray spectrum and the passage of electrons through gases.[13]

11. Hund's scientific diary 23 June 1922, Hund Papers.

12. Kramers (1923).

13. Franck himself described his new interpretation to Bohr as "pretty much sug-

The point of departure for Franck's integration was the most problematic and tentative aspect of Kramers's approach: in 1915 William Duane and Franklin L. Hunt had already established that the continuous X-ray spectrum breaks off at a maximum frequency v_{max}, which is determined by the kinetic energy of the scattering electron according to the Planck relation ($v_{max} = E_{kin}/h$).[14] While the existence of such a threshold made it obvious that *Bremsstrahlung* had to be understood as the result of a quantum process, it posed considerable problems for the correspondence approach. The Fourier-representation of an aperiodic process does not vanish at a maximum frequency but ranges from frequency 0 to ∞. Kramers had been at a loss, mathematically and physically, on how to deal with the apparent incompatibility of the observed spectrum and the Fourier-representation. Without any self-consistent justification, he simply cut off the spectrum obtained by the correspondence approach at the maximum frequency and assumed that the scattering electron would not radiate with higher frequencies. An electron that was bound by the atom, however, would still emit radiation in the form of characteristic X-ray lines and, as Kramers speculated, the intensity of these lines would be equal to the integral intensity of the cut-off part of the spectrum.

It was precisely this speculation and with it the question whether there was any significance to the cut-off part that became the source for Franck's new interpretation. When he considered the problem of the Duane-Hunt-threshold and the correspondence approach in connection with Ramsauer's results, Franck developed Kramers's idea in a different direction. For noble gases, he knew, the binding of the electron was out of the question. The cut-off part was thus associated with electrons that were scattered but did not emit radiation. Taking a huge step, Franck connected this assessment with Ramsauer's experiments and convinced himself that the absence of radiation meant nothing but the absence of interaction in general:

gested" by Kramers's work. Franck to Bohr 23 December 1922, BSC (2.4). As we will see, neither Franck's physical hypothesis nor Hund's technical elaboration was a mere duplication of Kramers's work.

14. Duane and Hunt (1915).

Electrons associated with the cut-off part do not radiate *and* pass the atom without being deflected:

> We are now certain that electrons ... really pass the atoms without deflection and we believe that this is even a requirement of quantum theory as soon as electrons have low velocities ... . In my opinion it is essential that the process occurs as soon as an electron would have to radiate more energy *quantenmäßig* [quantumlike] upon its entrance into the atom, than it possesses.[15]

Based on this assumption, the velocity dependence in Ramsauer's experiments received a new, strikingly simple explanation: For slow electrons, the cut-off part of the spectrum becomes larger and larger so that the atom becomes transparent for more and more electrons.

Within the analysis of the adaptation of the correspondence principle, Franck's visit to Copenhagen gives a first answer to the question of how physicists came to work with the principle. For Franck, the dialogue with Bohr and Kramers was the key for the adaptation within his work. That such a transfer of the principle would lead to anything like a new explanation of Ramsauer's puzzling results was not obvious. Rather, it took the tentative adaptation of a new approach to scattering and the combination of puzzling facts like the Duane-Hunt-threshold and Ramsauer's results to formulate Franck's no-interaction interpretation of the effect.

3. Hund's and Franck's work with the correspondence principle: Problems and adaptations

Upon Franck's return from Copenhagen, Franck and Hund engaged more deeply in the extension of the correspondence principle to scattering. Following the new hypothesis, Franck assumed that this work would be mostly technical. Hund would more or less redo Kramers's Fourier-analysis, map the Fourier-representation of the scattering electron onto the radiation spectrum and make the cut-off at the maximum frequency to substantiate Franck's new inter-

15. Franck to Bohr 23 December 1922, BSC (2.4).

pretation. While Franck had not seen Kramers's extensive calculations, he assumed that this was a standard task for a theoretician.[16] Things turned out quite differently. Hund recognized that the new approach entailed many more conceptual intricacies than Franck had imagined. While these issues could be resolved by tweaking the principle, Hund nonetheless obtained untenable results. Trying to save the correspondence approach from its apparent failure, Franck's and Hund's work came to a close with a reinterpretation of the scattering process, which took a first step towards a description of scattering in quantum-theoretical terms.

Making himself familiar with Franck's idea, Hund studied Bohr's formulation of the correspondence principle and tried to extract a prescription for his own work. For the atom, he understood, the principle associated the frequencies and Fourier-coefficients of the stationary states with the frequency and the probability of a quantum transition.[17] In order to extend the principle to scattering, Hund realized, he had to think about scattering in terms of the Bohr model, i.e., to identify the initial and the final orbit of the scattering electron and the transitions between them. Only then could he sensibly think about a correspondence relation between the Fourier-representation of the states and the radiation frequency and transition probability.

This first step was already quite problematic, as aperiodic motions could not be described within the framework of Bohr's quantum theory of multiply periodic systems. The lack of a mechanical framework, however, did not put an end to Hund's attempt to use the correspondence principle. He intuitively identified the electron approaching the atom with the initial state and – vice versa – the electron that had left the atom with the final state. By definition, the transition had to take place somewhere between these two states and would be connected with the deflection of the electron.

With this intuitive adaptation of the Bohr model, Hund approached the second item on the agenda. Applying the correspond-

16. Franck to Bohr 23 December 1922, BSC (2.4).

17. Hund (1922), p. 43. Hund's understanding of the principle was in accordance with Bohr's formulation of the principle and did not change fundamentally in the final paper, Hund (1923), pp. 250-251.

ence principle, he immediately realized that the newly defined "initial and final orbits are straight, they have no frequencies."[18] As such, they could not possibly be connected with a continuous radiation spectrum. In other words, the new description of scattering was incompatible with the original correspondence principle. To resolve the conflict, Hund adapted the correspondence principle by "let[ting] the frequency of the classically continued initial and final trajectory relate to the emitted frequencies."[19] The "classically continued trajectories" were of course nothing but the hyperbolic curves described by the electron during its deflection. Its Fourier-representation would lead to the desired continuous spectrum. With this second adaptation, Hund reached a position that allowed him to follow Franck's hypothesis: He could connect the Fourier-representation of the hyperbolic trajectory with the continuous radiation spectrum, cut it off at the maximum frequency and associate the cut-off part with electrons that pass the atom without deflection.

Hund's approach to the correspondence principle is typical of this time. Following the correspondence approach, most physicists recognized a tension between its original formulation and the problem they were addressing. Reacting to this tension, they tweaked the principle in one way or another. In many cases this *adaptive reformulation* left the core idea of the principle intact: The Fourier-representation of the radiating system was mapped onto the radiation spectrum. Tweaking the principle, physicists adapted other parts of it. In Hund's case, it was the reference system of the principle: the relevant Fourier-representation was no longer associated with the initial or the final state of the system. Instead the "frequencies of the classically continued initial and final trajectory" were associated with the deflection in the classical description or in other words with the quantum transitions.

Without reflecting on the significance of his adaptation, Hund moved on to the Fourier-analysis of the electron's trajectory and discovered a more striking problem. Before he made any extended calculations, Hund estimated the dominant frequencies in the spec-

18. Hund (1922), p. 43.
19. Hund (1922), p. 43.

trum and came to the conclusion that the energies *hv* were "considerably larger" than the kinetic energy of the electrons in Ramsauer's experiment.[20] Because most of the scattering electrons had to radiate with these frequencies, Hund arrived at a drastic conclusion: The atom was already transparent for velocities of several volts, "of which one knows enough experiments that indicate the reflection of electrons." In other words Franck's "correspondence approach appears to be impossible."[21]

This failure of the correspondence approach did not mean, however, that Franck's basic idea had to be abandoned. Dropping the quantum approach, Hund considered the classical radiation of the scattering electron and formulated a "modified Franckian conjecture": A slow electron, he argued, would not be able to leave the atom again if it lost too much energy during a collision. Without the possibility to be bound to the atom, it would crash into the nucleus. To prevent such a crash, Hund assumed, the electrons in question had to cease radiating and according to Franck's hypothesis they also had to stop interacting with the atom.[22]

Talking to Franck about the failure of the correspondence approach and his modification, Hund had a confrontation with Franck on the feasibility of the two explanations.[23] We do not know how the argument was eventually resolved. Hund stuck to his interpretation and handed in his dissertation at the university with Franck's approval. At the same time, Franck did not take Hund's counterargument to be the final word on the subject. He and Born concluded that Hund's dissertation showed that a classical explanation was untenable but it only gave a "first overview on the possibility" of a quantum-theoretical explanation.[24]

To save the correspondence approach, Franck developed "a new, very plausible conception," as Hund noted into his diary, "in keep-

20. Hund's scientific diary 20 October 1922 (Hund Papers). For Hund's estimate see Hund (1922), p. 44.

21.Hund (1922), p. 44.

22. Hund (1922), p. 42.

23. Hund's scientific diary 24 October 1922, Hund Papers.

24. See Born's evaluation of Hund's dissertation in the Promotionszulassungen der mathematisch-naturwissenschaftlichen Fakultät, UAG.

ing with the correspondence principle."[25] In doing so Franck proposed a quantum-theoretical descriptions of scattering that was built on the idea of transitions between different states but did not involve the classical description of scattering. This new interpretation extended Bohr's way of thinking about the correspondence principle and the quantum-classical divide. According to Bohr, the classical description of radiation and its quantum-theoretical counterpart were irrevocably divided. This conceptual divide, Bohr had stressed time and again, was not overcome by the correspondence principle, which expressed a formal analogy between the conceptually distinct theories. Trying to resolve Hund's counter argument, Franck adopted a similar point of view. He stressed that the observed spectrum was a manifestation of a quantum process and had to be described in quantum terms. The classical spectrum of the correspondence approach only provided an approximate representation of the observed spectrum in the absence of a detailed quantum description. This did not mean, however, that the underlying classical and quantum processes were connected on a conceptual level.[26]

Decoupling the classical from the quantum-theoretical spectrum in this way, Franck extended the divide. The classical description of scattering, he assumed, had nothing to do with the actual motion. In a future quantum theory, this motion would be described as a transition process from one straight line to another and the atomic force field no longer curved the path of the electron but created a probability for the transition:

> During the collision of an electron with an atom a *transition probability* is created under the influence of the atomic force field [for a transition] from a straight orbit, on which the electron arrives, to another orbit of lower energy. The new orbit also is a straight line.[27]

In this new description the correspondence principle could be applied without running into Hund's counterargument. The classical

25. Hund's scientific diary 29 October 1922, Hund Papers.
26. Hund (1923), p. 254.
27. Hund (1923), p. 262, emphasis in the original.

trajectory of the electron was no longer part of the new quantum description and therefore the association of the spectral frequencies with the angles of deflection was no longer in play. Instead the deflection of the electrons into various angles was governed by a probability distribution, for which the classical spectrum provided a first approximation.

While they recognized that the new description led to additional problems,[28] Franck's and Hund's work came to a close with Franck's new conception of scattering. Their work was received quite positively within the quantum community. Kramers hoped that Franck's "nice idea, that radiationless collision means interactionless collision in general, should be confirmed,"[29] while Bohr took it as another sign of the inadequacy of a spatio-temporal description of transition processes.[30] A constructive effort to develop a quantum theory of scattering, however, was only undertaken by Born and Jordan in the summer of 1925. While Heisenberg and the Copenhagen community worked on the sharpening of the correspondence principle in spectroscopy and ended up with *Umdeutung*, Born and Jordan followed up on the correspondence approach to scattering and formulated an account of aperiodic processes and a general "correspondence principle of motion."[31]

4. Conclusion

Franck's and Hund's work on scattering was peripheral for the development of matrix mechanics or a successful quantum theory of collision processes. Nonetheless, their work is highly significant for

28. Observing that the intensity remained finite for zero-frequency radiation, Hund realized that the transition probability became infinite. Thereby he encountered the infrared divergence for the first time. The issue was resolved by Born's new assistant, Werner Heisenberg, who suggested that electrons could radiate infinitely many times for frequency zero during one transition. Both the problem and its solution were ahead of their time; they only became important in the development of QED. See Blum (2014).

29. Kramers to Franck 8 January 1923, Franck Papers, Box 4, Folder 9.

30. Darrigol (1992), p. 251.

31. Born and Jordan (1925).

understanding the correspondence principle and its practice. It illustrates how the principle was dispersed and adapted within the old quantum theory: Driven by the challenging problems rather than a search for an overarching quantum theory, Franck and Hund – and with them many other physicists – integrated the principle into their research. They recognized the principle as a resource for the solution of their problems and adopted it as a research tool. The core of this tool – the connection of the Fourier-series of the radiating system with the emitted spectrum – was immensely stable and at the same time flexible enough to be extended to phenomena as diverse as spectroscopy, dispersion and scattering.

Dealing with these phenomena and physical models, which were vastly different from the Bohr model and atomic spectra, did not allow physicists to work with the principle as a ready-made tool in its proper context. Rather, practising the correspondence principle meant adapting it to the structure of the problems at hand. Whether physicists attempted to account for isolated problems or whether they tried to develop a new theory like matrix mechanics, this process of *adaptive reformulation* played an important role. In the long run, it led to a new understanding of challenging phenomena and a "sharpening" of the correspondence principle.

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Franck Papers	James Franck Papers, Special Collections Research Center, University of Chicago Library
Hund Papers	Nachlass Friedrich Hund, Handschriften und Nachlässe,

	Niedersächische Staats- und Universitätsbibliothek Göttingen
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